Secure computation

Lecture 6

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Modeling Adversaries

- Adversarial behavior
  - **Semi-honest:** follows the protocol specification
    - Tries to learn more than allowed by inspecting transcript
  - **Malicious:** follows any arbitrary strategy
- Adversarial power
  - **Polynomial-time**
  - **Computationally unbounded:** information-theoretic security
  - (based on slides of Yehuda Lindell)

Modeling Adversaries

- Corruption strategy
  - **Static:** the set of corrupted parties is fixed before the execution begins
  - **Adaptive:** the adversary can corrupt parties during the execution, based on what has happened
    - Models modern "hacking"
    - In general, **much harder!**

Execution Setting

- Stand-alone
  - Consider a single protocol execution only (or that only a single execution is under attack)
- Concurrent general composition
  - Arbitrary protocols executed concurrently
  - Realistic setting, very important model
- Stand-alone vs composition
  - **Stand-alone:** a good place to start studying secure computation, techniques and tools are helpful
  - **Composition:** true goal for constructions

Preliminaries

- Notations:
  - Security parameter **n**
  - We wish security to hold for all inputs of all lengths, as long as **n** is large enough

- Function μ is negligible: if for every polynomial $p(\cdot)$ there exists an N such that for all $n>N$ we have $\mu(n) < 1/p(n)$

Preliminaries

- Probability ensemble X={X(a,n)}
  - Infinite series, indexed by a string **a** and natural **n**

  - Each **X**(**a,n**) is a random variable
    - In our context: the output of a protocol execution with input **a** and security parameter **n**
    - Probability space: randomness of parties

Preliminaries

- Computational indistinguishability $X \approx Y$
  - For every (non-uniform) polynomial-time distinguisher D there exists a negligible function μ such that for every **a** and all large enough **n**'s:

$$|\Pr[D(X(a,n))=1] - \Pr[D(Y(a,n))=1]| < \mu(n)$$

Notation

- Functionality
  - $\mathbf{f}=(\mathbf{f}_1,\mathbf{f}_2)$: for input vector $\mathbf{x}$, each $f_i(\mathbf{x})$ is a random variable (for probabilistic functionalities)
  - Party $\mathbf{P}_i$ receives $\mathbf{f}_i$
  - We denote $(\mathbf{x},\mathbf{y}) \rightarrow (\mathbf{f}_1(\mathbf{x},\mathbf{y}),\mathbf{f}_2(\mathbf{x},\mathbf{y}))$

Semi-Honest Adversaries

- Simulation:
  - Given input and output, can generate the adversary's view of a protocol execution
  - Important: since parties follow protocol, the inputs are **well defined**

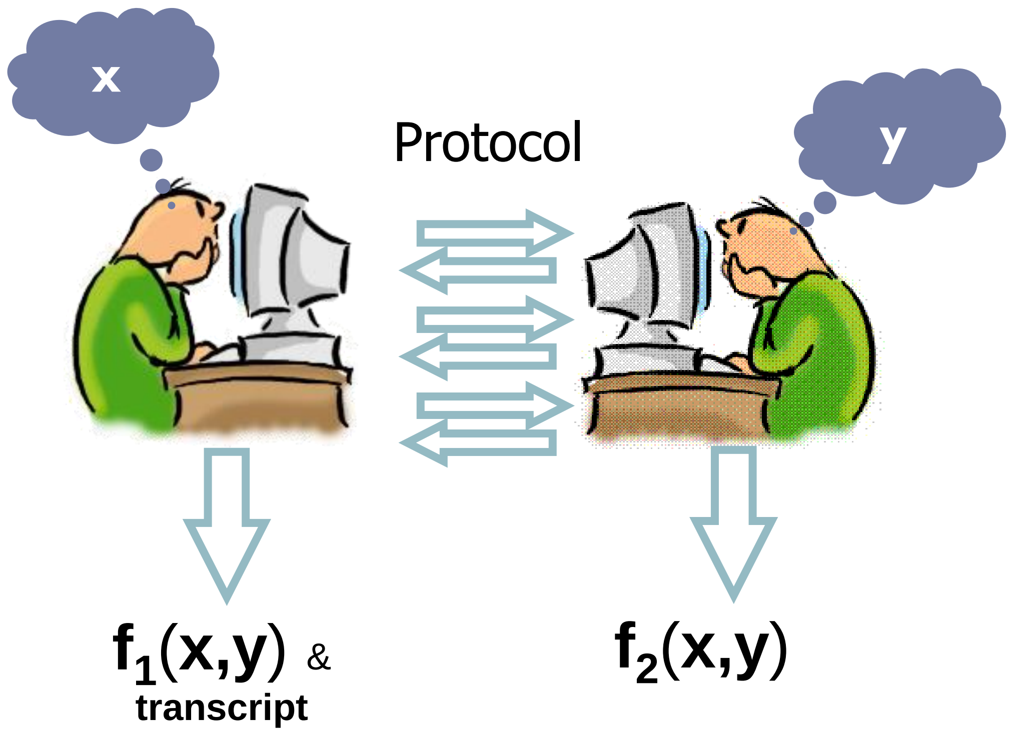
Security definition: Semi-Honest Adversaries

- $\forall$ semi-honest adversary A controlling P1, $\exists$ simulator S1 such that for every pair of inputs (x,y), the following are *computationally indistinguishable*
  - The output of **A**, and the output of the honest party P2 after a protocol execution
  - The output of **S1** given $\mathbf{x_1}$ and $\mathbf{f_1(x,y)}$, and the value $\mathbf{f_2(x,y)}$

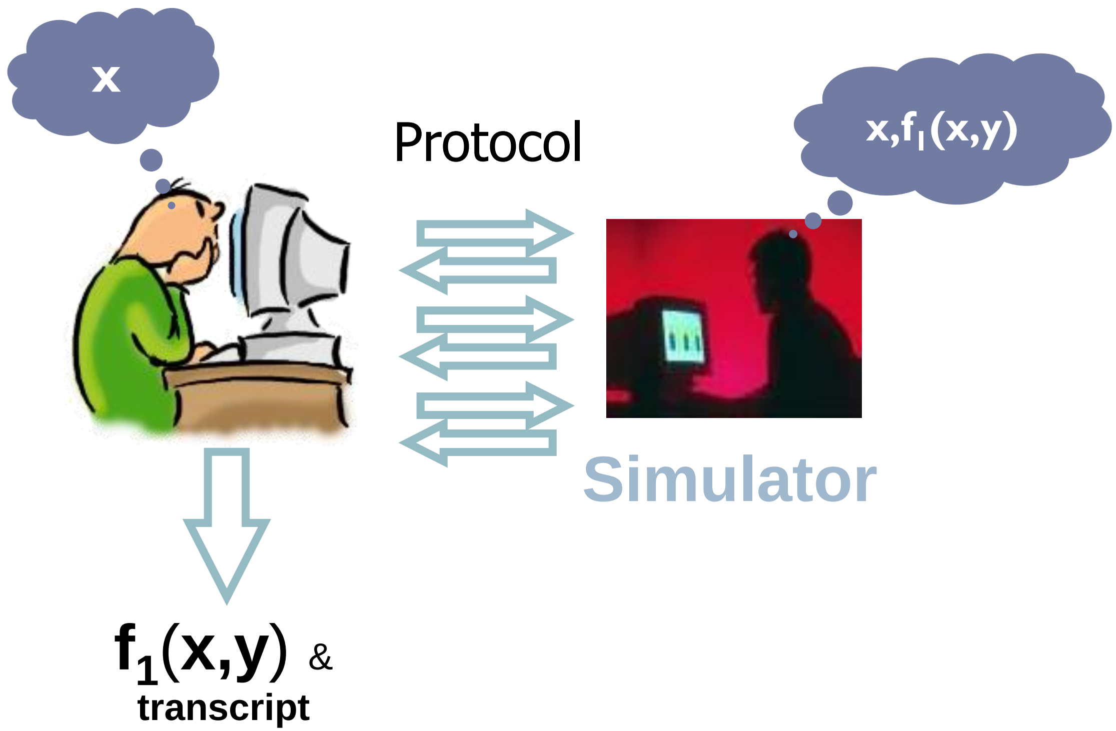
Similarly, $\forall$semi-honest A controlling P2, $\exists$ S2, such that $\forall$ inputs (x,y), the following are *computationally indistinguishable*

- The output of **A**, and the output of the honest party P1 after a protocol execution
- The output of **S2** given $\mathbf{x_2}$ and $\mathbf{f_2(x,y)}$, and the value $\mathbf{f_1(x,y)}$

Semi-Honest Adversaries



Semi-Honest Adversaries



Properties

- Correctness, independence of inputs, fairness are all non-issues in the semi-honest model

- Why is privacy guaranteed by this definition?
  - If the adversary can compute something after a real protocol execution, it can compute it just from the input/output
  - The adversary's view in an execution can be generated from the input and output only
  - Very similar to zero-knowledge

Joint Distribution

- A crucial point: need to consider the joint distribution of adversary's output and honest parties' output

- In the definition:
  - We compare the distribution of all inputs and outputs together with the adversary's output

Joint Distribution

- Example:
  - **Functionality: A** outputs random bit, **B** outputs nothing
    - **B** should clearly not learn **A**'s output bit
  - **Protocol: A** chooses a random bit, outputs it, and sends the bit to **B** (who ignores it)
- This protocol is clearly insecure.
  - But it is simulatable when separately looking at the distribution of B's view and actual outputs
  - However, it is not simulatable when working according to the definition

Deterministic Functionalities

- In the case of deterministic functionalities, the outputs are fully determined by the inputs
- It suffices to separately prove
  - Correctness
  - Simulation: show that can generate view of semi-honest adversary (corrupted parties' view), given inputs and outputs only

    - In other words…

Deterministic Functionalities

- Separately prove the following two statements
  - The output of the protocol is indistinguishable from the output of the functionality
  - There exists a simulator S1 such that for any adversary A controlling P1, the output of **A**, and the output of **S1** given $\mathbf{x}_1$ and $\mathbf{f}_1(\mathbf{x})$, are indistinguishable.
  - Similarly, that there exists a simulator S2 such that for any adversary A controlling P2, the output of **A**, and the output of **S2** given $\mathbf{x}_2$ and $\mathbf{f}_2(\mathbf{x})$, are indistinguishable.

Malicious Adversaries

- First attempt: require the existence of a simulator that generates the adversary's view given the inputs/outputs of the corrupted party

- Problem: what are the inputs used by the adversary?
  - They are not necessarily those written on the input tape
  - They are not explicit: the adversary doesn't run the protocol but arbitrary code
  - For example, in the Bellare-Micali OT protocol, a malicious server can send two random messages without knowing what they encrypt

The Ideal/Real Paradigm

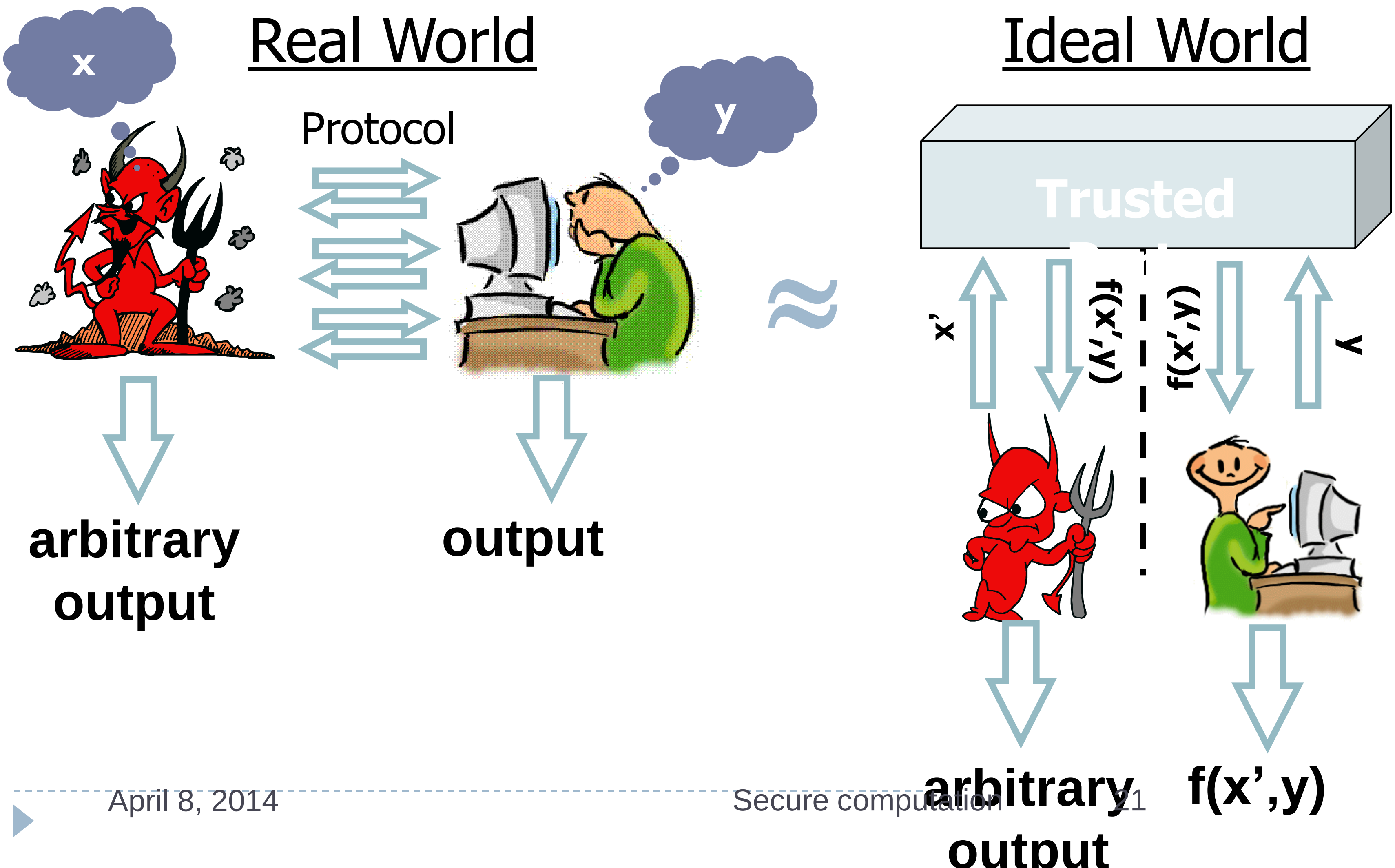
- What is the best we could hope for?
  - An incorruptible trusted party
  - All parties send inputs to trusted party (over perfectly secure communication lines)
  - Trusted party computes output
  - Trusted party sends each party its output (over perfectly secure communication lines)
  - This is an **ideal world**

- What can an adversary do?
  - Just choose its input…

The Ideal/Real Paradigm

- We would like our real protocol to behave like the ideal world
- Formalizing this notion:
  - For *every adversary **A*** attacking the real protocol, *there exists an adversary **S*** in the ideal model such that the output distributions (of all parties) are **computationally indistinguishable**
  - **S** simulates a real protocol execution while interacting in the ideal world
  - Here we always look at the joint output distribution

The Ideal/Real Paradigm



"Formal" Security Definition

- Protocol π securely computes a function f if:
  - For every non-uniform polynomial-time real-model adversary **A**, there exists a non-uniform polynomial-time ideal-model adversary **S**, such that for all input vectors and auxiliary inputs:
  - the joint outputs of **A** and the honest party in a real execution of π are indistinguishable from the joint outputs of **S** and the honest party in an ideal execution where the trusted party computes **f**

Properties

- The following properties hold
  - Privacy: from adversary's outputs
  - Correctness: from honest party's output
  - Independence of inputs: from ideal execution
  - Fairness and guaranteed output delivery: from ideal execution

Relaxing the Ideal Model

- In some cases, this ideal model is too strong and cannot be achieved

- Fairness cannot be achieved in general without an honest majority
  - Consider two parties and consider removing the last message of the protocol execution
    - Works for coin tossing…

Relaxing the Ideal Model

- In order to model the case that fairness is not guaranteed, change the instructions of the trusted party in the ideal model:
  - Trusted party receives input from all parties
  - Trusted party sends corrupted party's output to adversary
  - Adversary says "continue" or "halt"
  - If "continue", trusted party sends output to honest party; else, it sends "abort"