Introduction to Cryptography

Lecture 5

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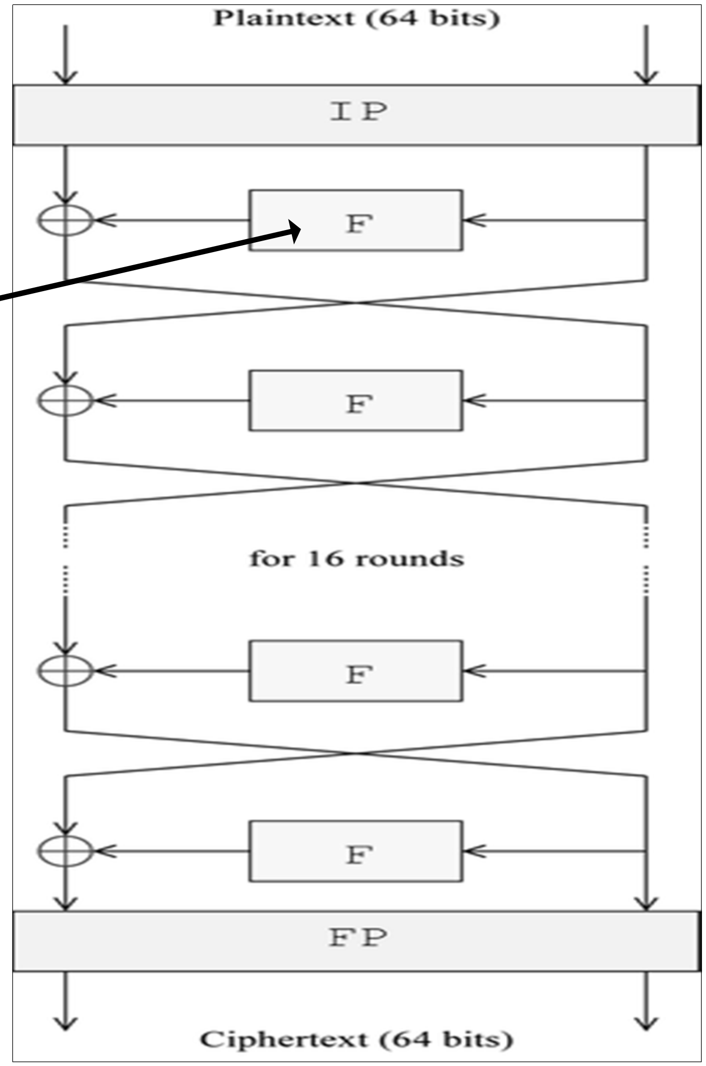
Differential Cryptanalysis

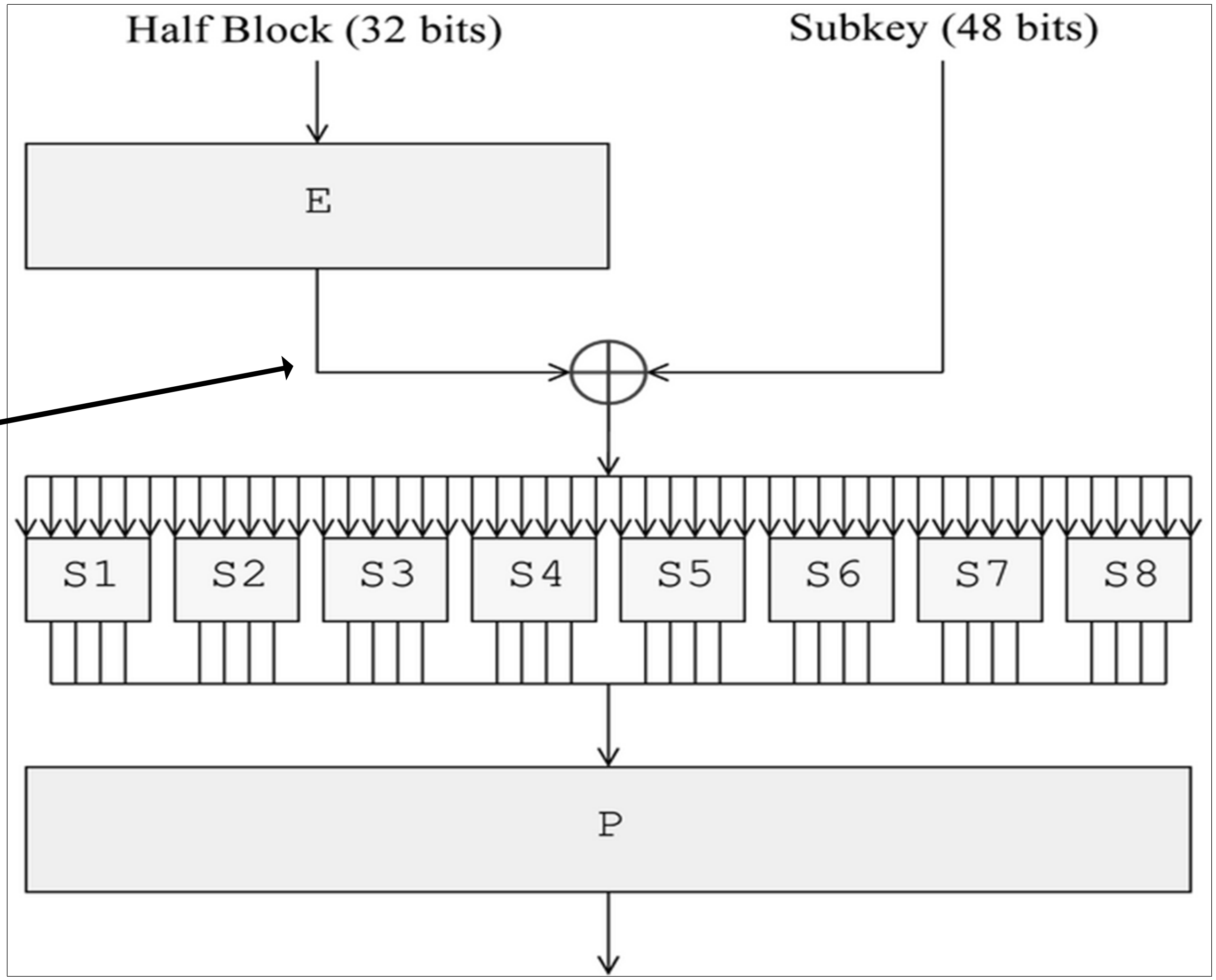
Differential Cryptanalysis of DES

DES diagram:

S-boxes



DES F functions



Expansion to 48 bits

Differential Cryptanalysis [Biham-Shamir 1990]

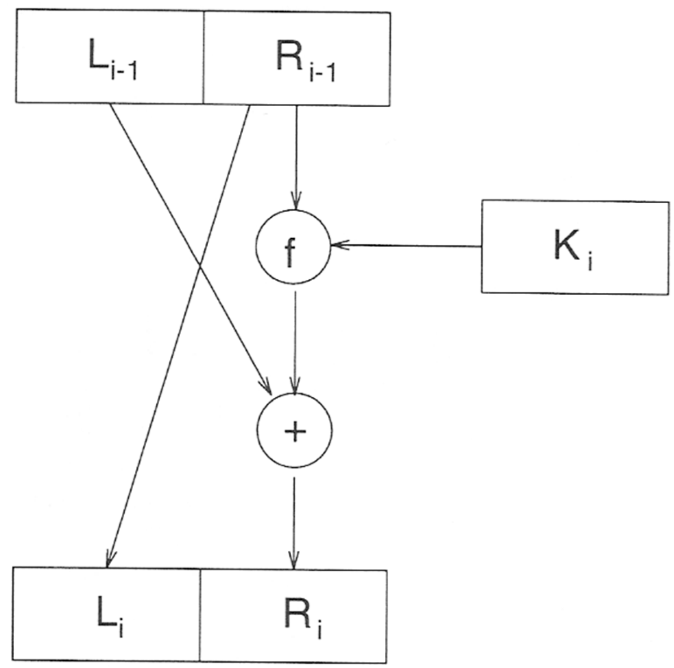
- The first attack to reduce the overhead of breaking DES to below exhaustive search
- Very powerful when applied to other encryption algorithms

- Depends on the structure of the encryption algorithm
- Observation: all operations except for the s-boxes are linear
- Linear operations:
  - $a = b \oplus c$
  - a = the bits of b in (a known) permuted order
- Linear relations can be exposed by solving a system of linear equations

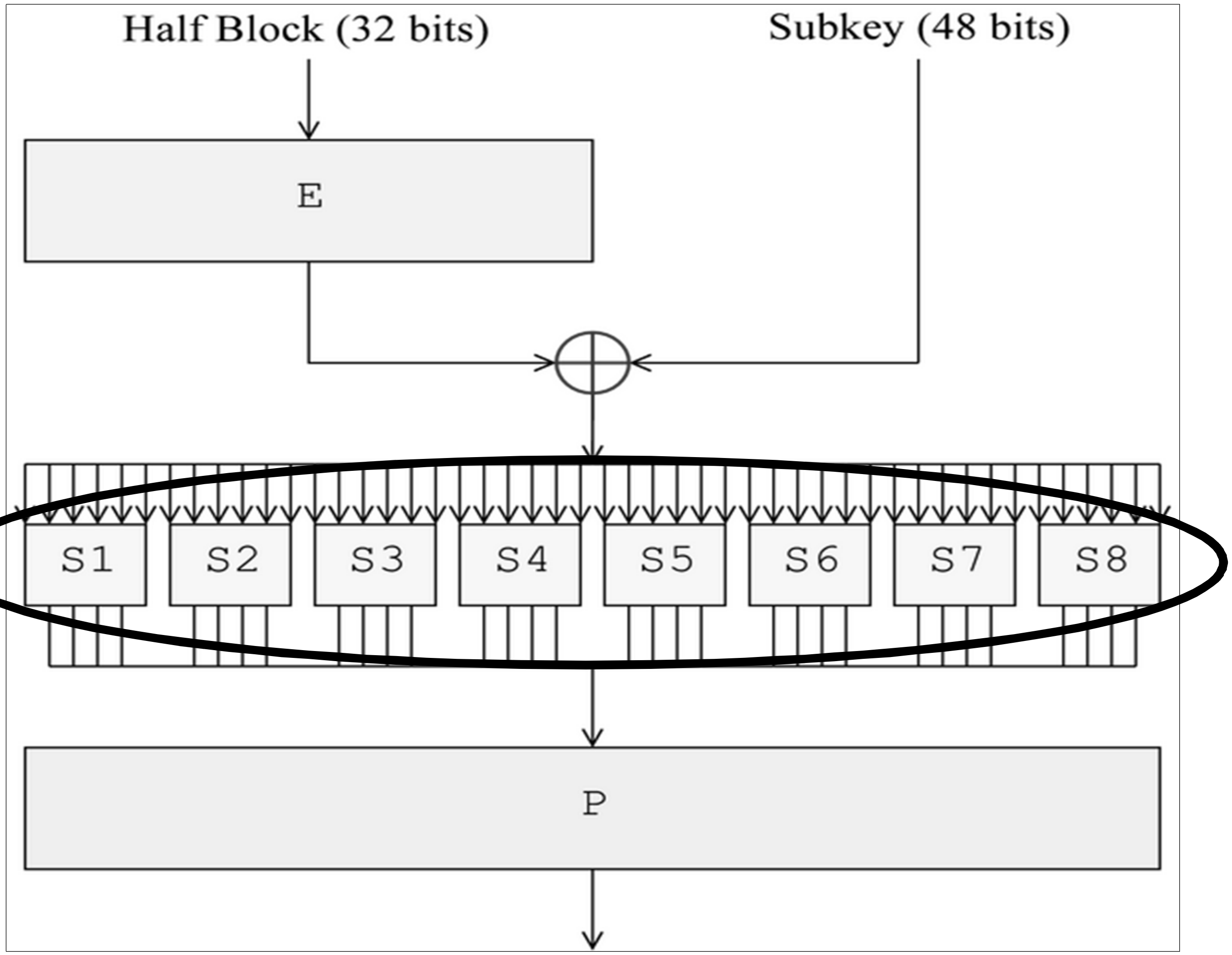
Is a Linear F in a Feistel Network secure?

- Suppose $F(R_{i-1},K_i) = R_{i-1} \oplus K_i$
  - Namely, F is linear
- Then $R_i = L_{i-1} \oplus R_{i-1} \oplus K_i$

  $L_i = R_{i-1}$
- Write L_{16}, R_{16} as linear functions of L_0, R_0 and K.
  - Given L_0R_0 and $L_{16}R_{16}$ Solve and find K.
- F must therefore be non-linear.
- F is the only source of non-linearity in DES.



DES F functions



Source of non-linearity

Differential Cryptanalysis

- The S-boxes are non-linear
- We study the differences between two encryptions of two different plaintexts

- Notation:
  - Denote two different plaintexts as P and P*
  - Their difference is $dP = P \oplus P^*$
  - Let X and X* be two intermediate values, for P and P*, respectively, in the encryption process.
  - Their difference is $dX = X \oplus X^*$
    - Namely, dX is always the result of two inputs

Differences and S-boxes

- S-box: a function (table) from 6 bit inputs to 4 bit output

- X and X* are inputs to the same S-box. We can compute their difference $dX = X \oplus X^*$.

- Y = S(X)
- When dX=0, X=X*, and therefore $Y=S(X)=S(X^*)=Y^*$, and dY=0.
- When dX≠0, X≠X* and we don't know dY for sure, but we can investigate its distribution.

- For example,

Distribution of Y' for S1

- dX=110100
- There are 2^6=64 input pairs with this difference, { (000000,110100), (000001,110101),…}
- For each pair we can compute the xor of outputs of S1
- E.g., S1(000000)=1110, S1(110100)=1001. dY=0111.
- Table of frequencies of each dY:

0000	0001	0010	0011	0100	0101	0110	0111
0	8	16	6	2	0	0	12
1000	1001	1010	1011	1100	1101	1110	1111
6	0	0	0	0	8	0	6

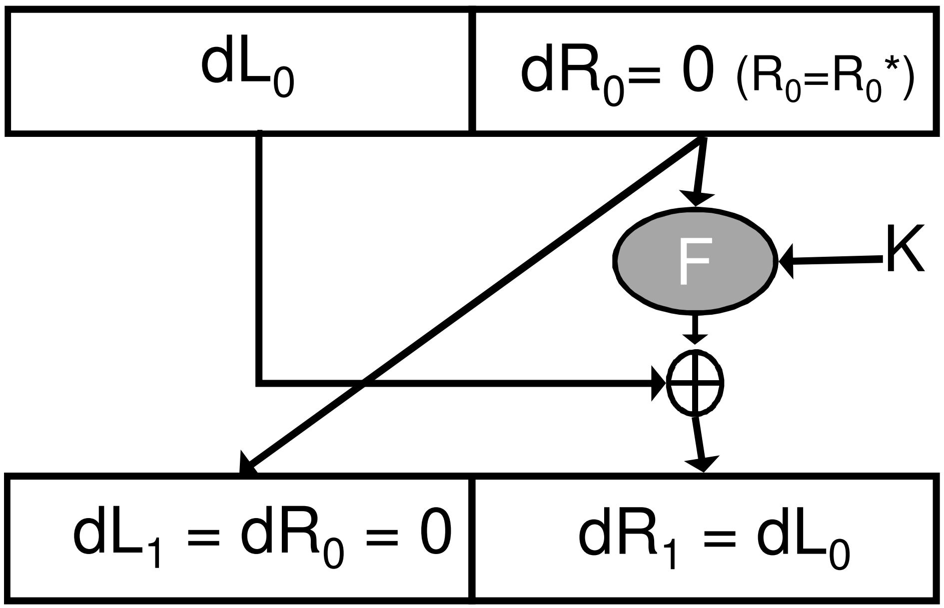
Differential Probabilities

- The probability of dX $\Rightarrow$ dY is the probability that a pair of inputs whose xor is dX, results in a pair of outputs whose xor is dY (for a given S-box).
- Namely, for dX=110100 these are the entries in the table divided by 64.

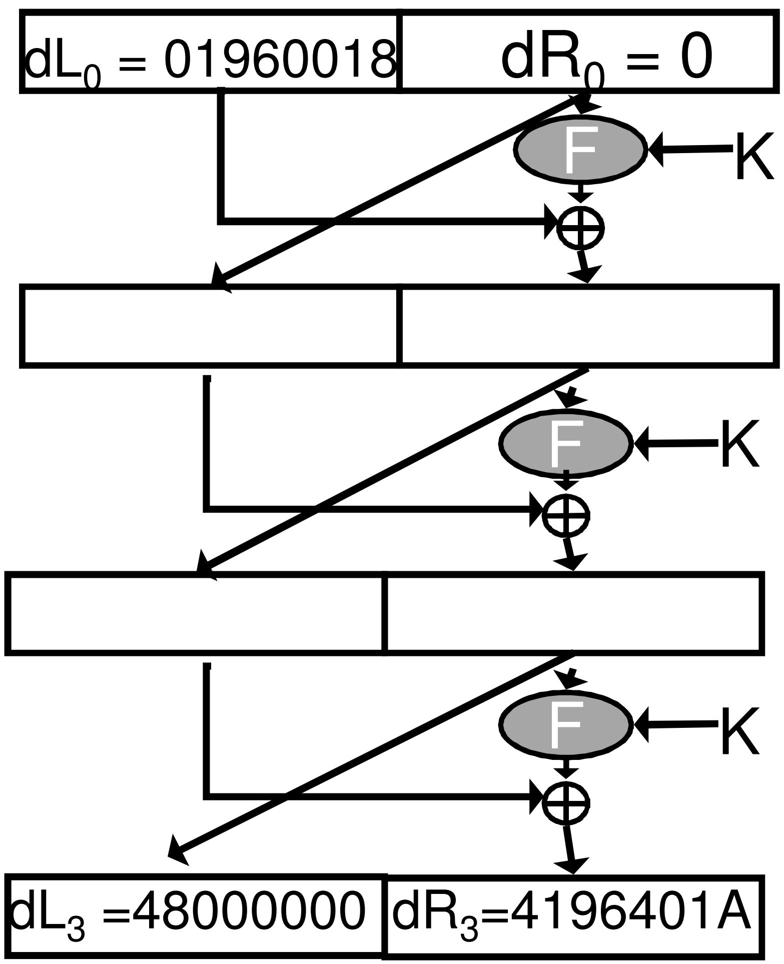
- Differential cryptanalysis uses entries with large values
  - dX=0 $\Rightarrow$ dY=0
  - Entries with value 16/64
  - (Recall that the outputs of the S-box are uniformly distributed, so the attacker gains a lot by looking at differentials rather than the original values.)

Warmup

Inputs: L_0R_0, $L_0^*R_0^*$,   s.t. $R_0=R_0^*$.
Namely, inputs whose xor is dL_0 0

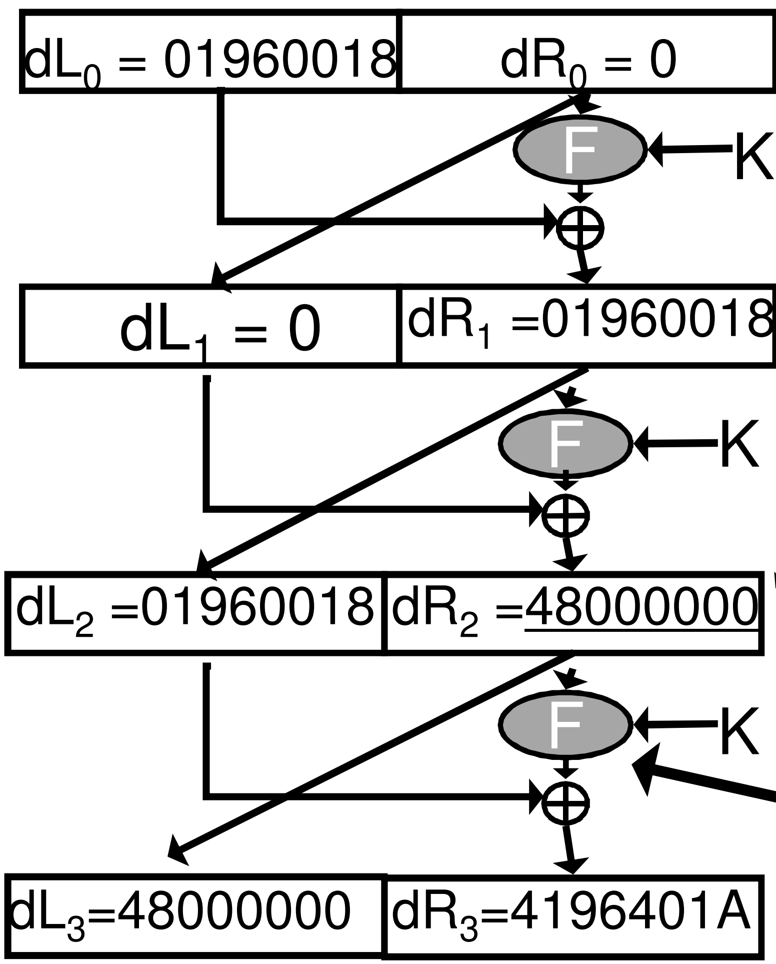


3 Round DES



The attacker knows the two plaintext/ciphertext pairs, and therefore also their differences

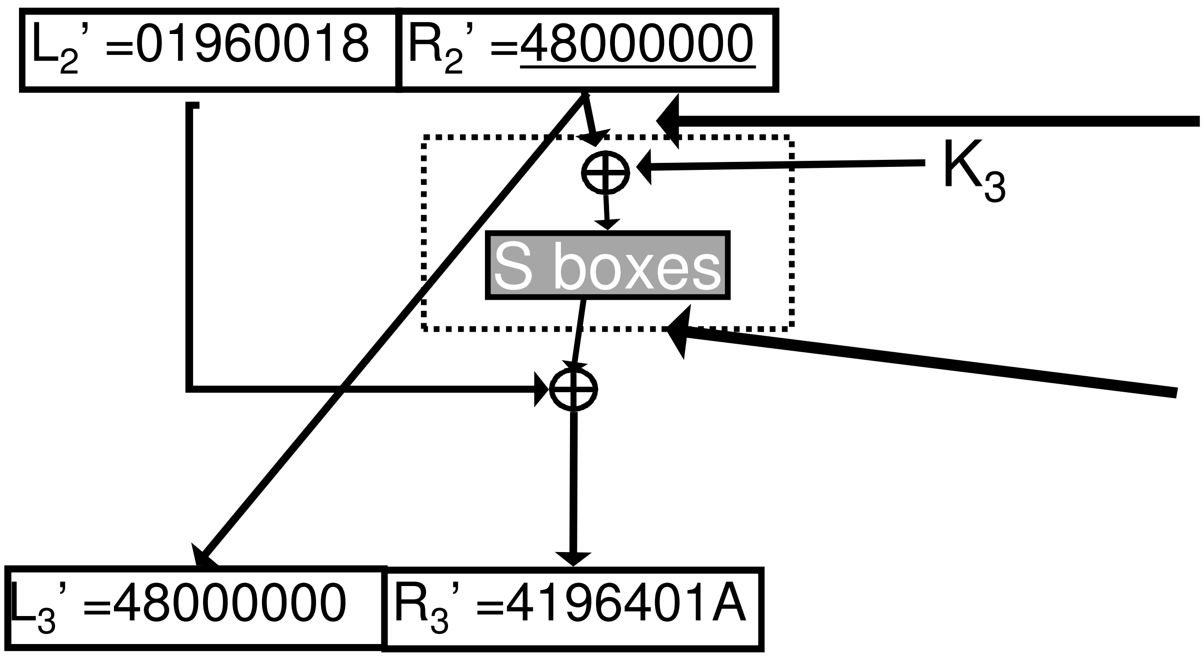
Intermediate differences equal to plaintext/ciphertext differences



Note that here the adversary also knows the actual two values

```
dF = 4196401A
⊕      01960018
=      40004002
```

Finding K



The actual two inputs to F are known

Output xor of F (i.e., S boxes) is 40004002

⇒Table enumerates options for the pairs of inputs to S box

Find which K_3 maps the inputs to an s-box input pair that results in the output pair!

DES with more than 3 rounds

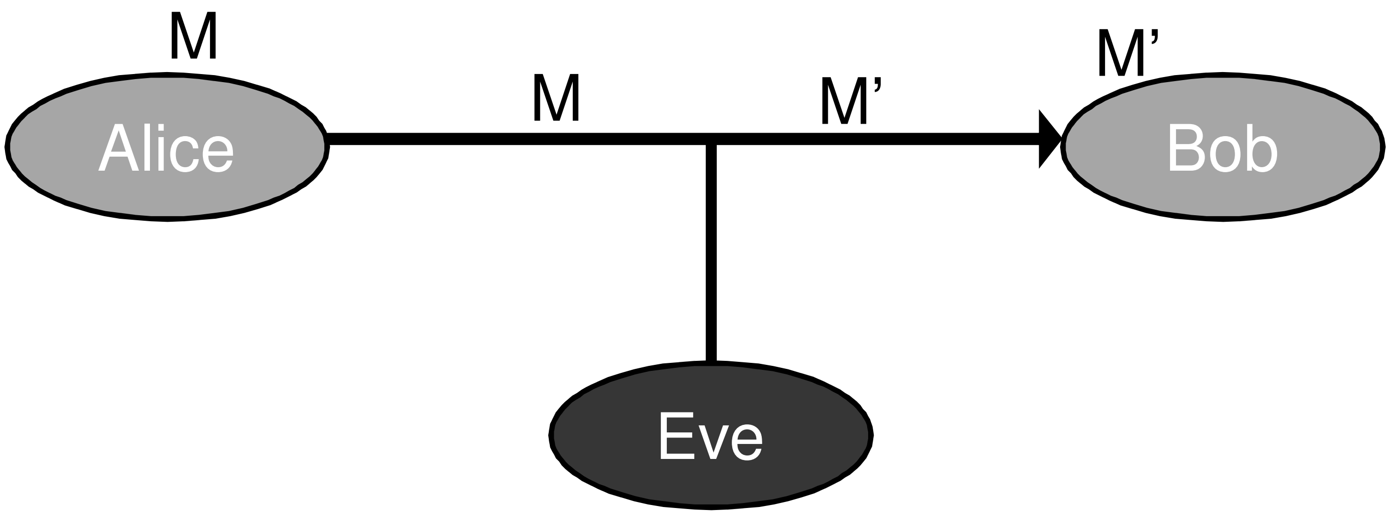
- Carefully choose pairs of plaintexts with specific xor, and determine xor of pairs of intermediate values at various rounds.
- E.g., if $dL_0=40080000_x$, $dR_0=04000000_x$
  Then, with probability ¼, $dL_3=04000000_x$, $dR_3=4008000_x$

- 8 round DES is broken given 2^{14} chosen plaintexts.
- 16 round DES is broken given 2^{47} chosen plaintexts...

Message Authentication

Data Integrity, Message Authentication

- Risk: an *active* adversary might change messages exchanged between Alice and Bob



• Authentication is orthogonal to secrecy. It is a relevant challenge regardless of whether encryption is applied.

One Time Pad

- OTP is a perfect cipher, yet provides no authentication
  - Plaintext $x_1x_2\ldots x_n$
  - Key $k_1k_2\ldots k_n$
  - Ciphertext $c_1=x_1\oplus k_1,\ c_2=x_2\oplus k_2,\ldots,c_n=x_n\oplus k_n$

- Adversary changes, e.g., c_2 to $1\oplus c_2$
- User decrypts $1\oplus x_2$

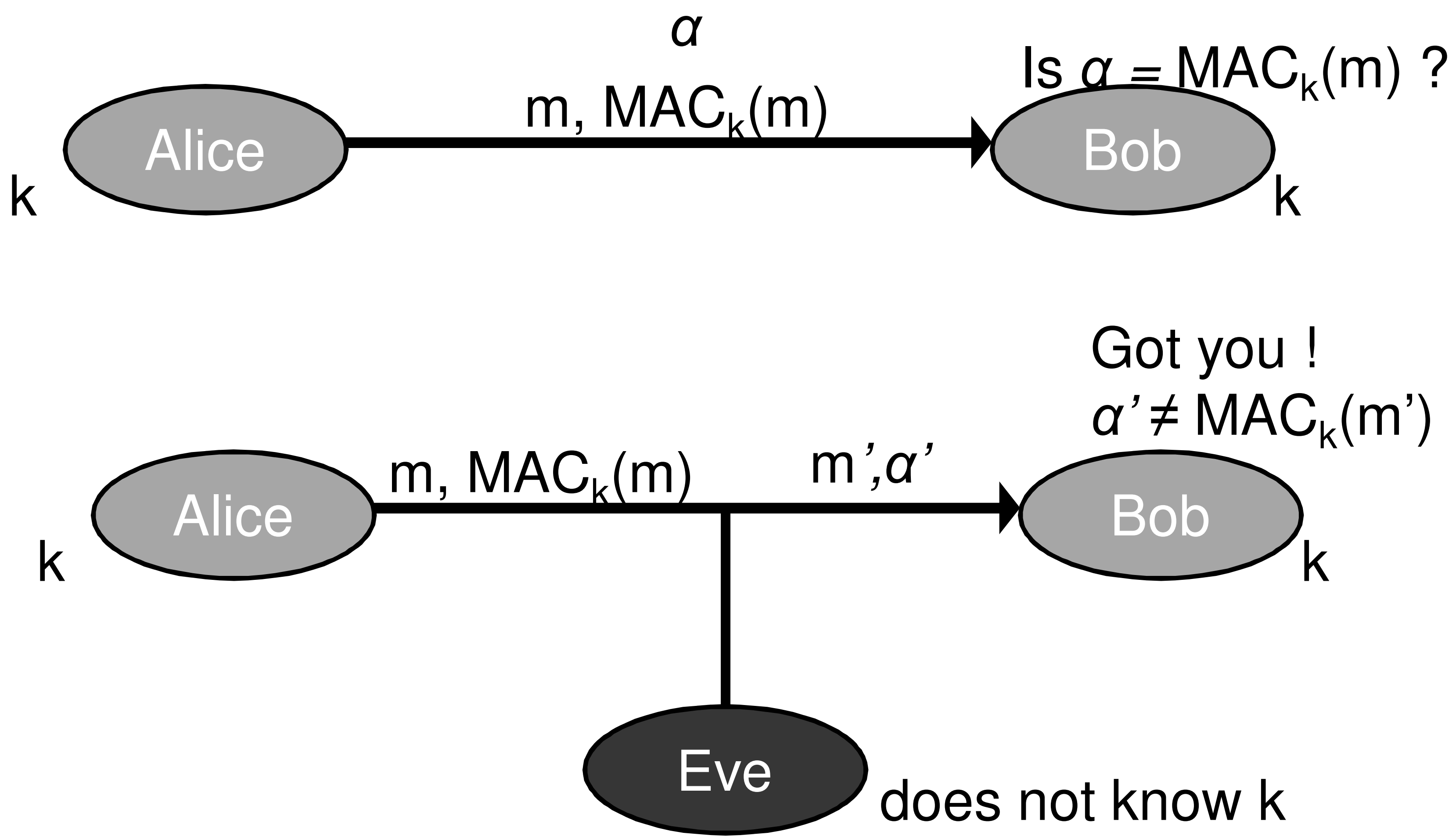
- Error-detection codes are insufficient. (For example, linear codes can be changed by the adversary, even if encrypted.)
  - They were not designed to withstand adversarial behavior.

The setting

- A random key K is shared between Alice and Bob.
- Authentication (tagging) algorithm:
  - Compute a Message Authentication Code: $\alpha = MAC_K(m)$.
  - Send m and α
- Verification algorithm: $V_K(m, \alpha)$. Output is a single bit.
  - $V_K(m, MAC_K(m)) = accept.$
  - For $\alpha \neq MAC_K(m)$, $V_K(m, \alpha) = reject.$

- How does $V_k(m)$ work?
  - Receiver knows k. Receives m and α.
  - Receiver uses k to compute $MAC_K(m)$.
  - $V_K(m, \alpha) = 1$ iff $MAC_K(m) = \alpha$.

Common Usage of MACs for message authentication



Requirements

- Security: The adversary,
  - Knows the MAC algorithm (but not *K*).
  - Is given many pairs $(m_i , MAC_K(m_i))$, where the m_i values might also be chosen by the adversary (chosen plaintext).
  - Cannot compute $(m, MAC_K(m))$ for any new m ($\forall i\ m \neq m_i$).
  - The adversary must not be able to compute $MAC_K(m)$ *even* for a message *m* which is "meaningless" (since we don't know the context of the attack).

- Efficiency: MAC output must be of fixed length, and as short as possible.
  - $\Rightarrow$ The MAC function is not 1-to-1.
  - $\Rightarrow$ An n bit MAC can be broken with prob. of at least 2^{-n}.

Constructing MACs

- Length of MAC output must be at least n bits, if we do not want the cheating probability to be greater than 2^{-n}

- Constructions of MACs
  - Based on block ciphers (CBC-MAC)

  or,
  - Based on hash functions
    - More efficient
    - At the time, encryption technology was controlled (export restricted) and it was preferable to use other means when possible.

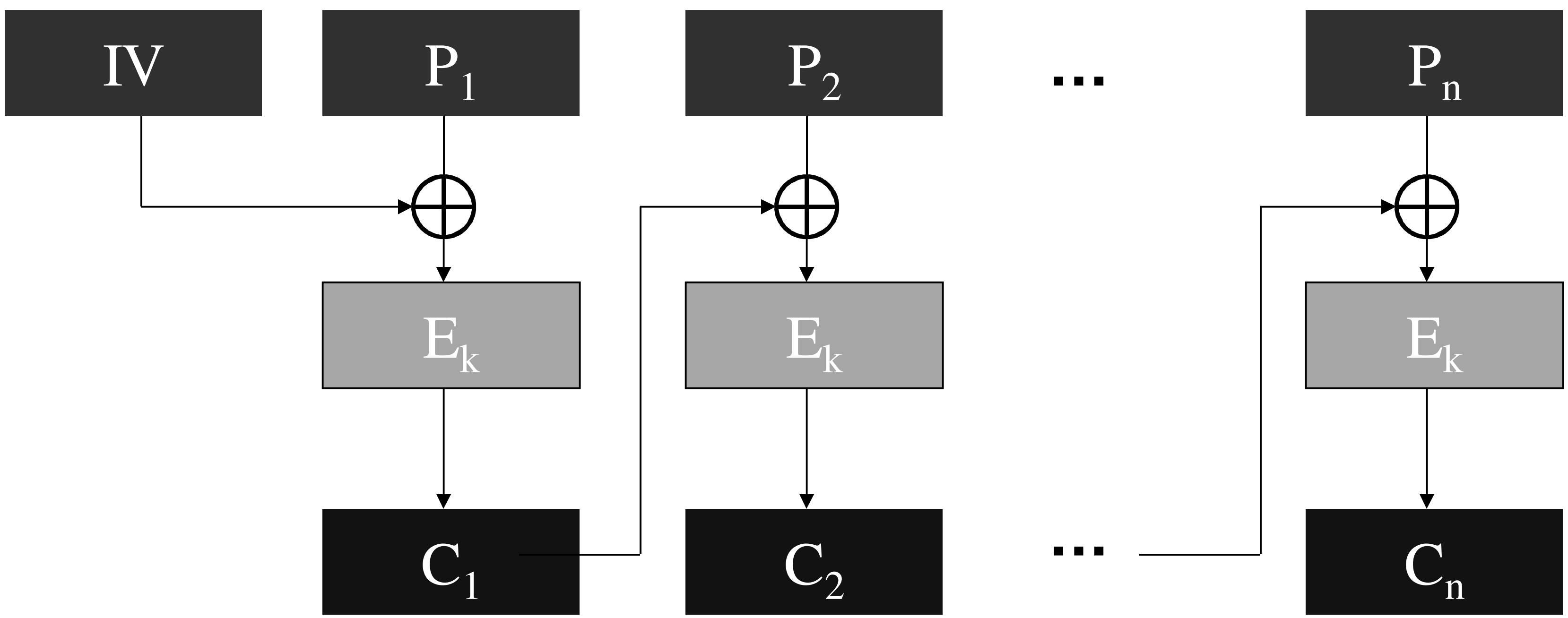
Definitions – security against chosen message attacks

- The authentication game
  - A secret key K is chosen at random.
  - The adversary can obtain the MAC $MAC_K(m)$ on any message m of its choice.
  - Let Q be the set of messages whose MACs were learned by the adversary.
  - At the end, the adversary outputs (m', α'), for an $m' \notin Q$.
  - The adversary succeeds if $V_K(m', \alpha') = accept$.

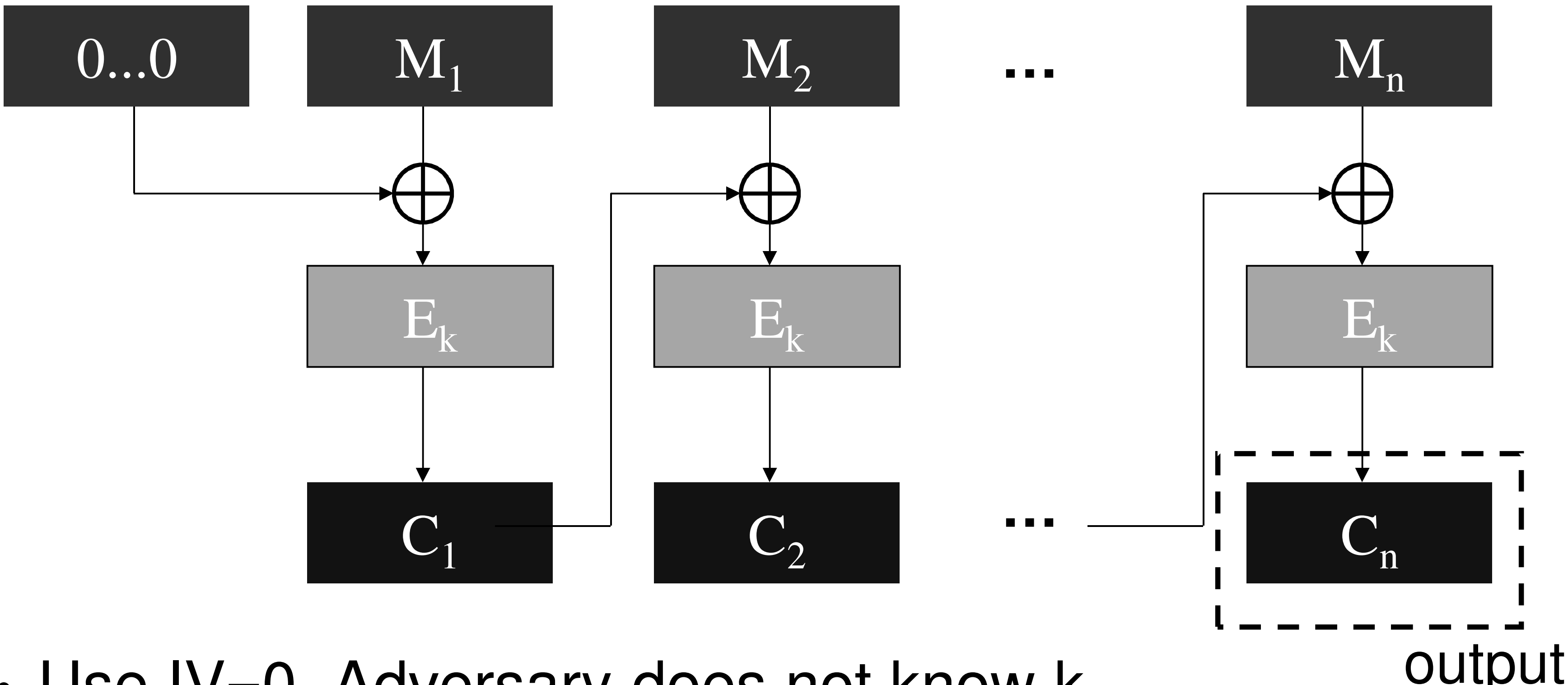
- A MAC is (t,ε)-secure if for every adversary A that runs at most t steps, the probability of success is at most ε.

CBC

- Reminder: CBC encryption
- Plaintext block is xored with previous ciphertext block



CBC MAC



- Use IV=0. Adversary does not know k.
- Encrypt M in CBC mode, using the MAC key. Discard $C_1,\ldots,C_{n-1}$ and define $MAC_K(M_1,\ldots,M_n)=C_n$.

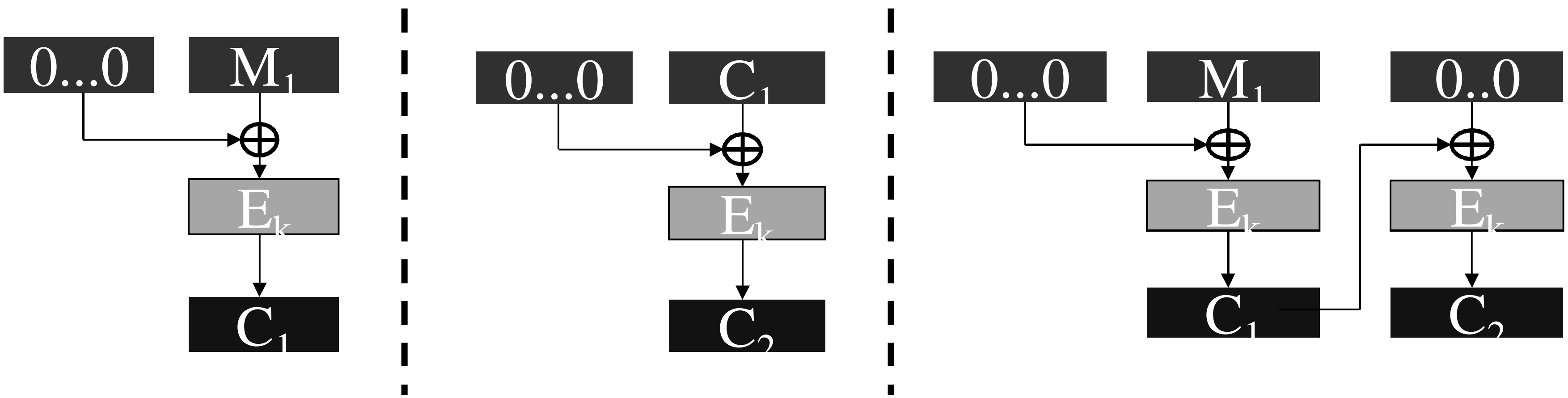
Security of CBC-MAC

- Claim: if E_K is pseudo-random then
  - CBC-MAC, applied to *fixed length messages*, is a pseudo-random function,
  - and is therefore a secure MAC (i.e., resilient to forgery).
- We will not prove this claim.

- But, CBC-MAC is insecure if *variable* length messages are allowed

Security of CBC-MAC

- Insecurity of CBC-MAC when applied to messages of variable length:
  - Get C_1 = CBC-$MAC_K(M_1) = E_K(0 \oplus M_1)$
  - Ask for MAC of C_1, i.e., C_2 = CBC-$MAC_K(C_1) = E_K(0 \oplus C_1)$
  - But, $E_K(C_1 \oplus 0) = E_K(\ E_K(0 \oplus M_1) \oplus 0)$ = CBC-$MAC_K(M_1 \mid 0)$
    - Can you show, for every n, a collision between two messages of lengths 1 and n+1?
    - It's known that CBC-MAC is secure if message space is prefix-free.



CBC-MAC for variable length messages

- Solution 1: The first block of the message is set to be its length. I.e., to authenticate $M_1,\ldots,M_n$, apply CBC-MAC to $(n,M_1,\ldots,M_n)$.
  – Works since now the message space is prefix-free.
  – Drawback: The message length (n) must be known in advance.

CBC-MAC for variable length messages

- "Solution 2": apply CBC-MAC to $(M_1,\ldots,M_n,n)$
  - Message length does not have to be known is advance
  - But, this scheme is broken (see, M. Bellare, J. Kilian, P. Rogaway, The Security of Cipher Block Chaining, 1984)
- Solution 3: (preferable)
  - Use a second key K'.
  - Compute $MAC_{K,K'}(M_1,\ldots,M_n) = E_{K'}(MAC_K(M_1,\ldots,M_n))$
  - Essentially the same overhead as CBC-MAC

Hash functions

- MACs can be constructed based on hash functions.
- A hash function $h:X \rightarrow Y$ maps long inputs to fixed size outputs. ($|X|>|Y|$)
- No secret key. The hash function algorithm is public.
- If $|X|>|Y|$ there are collisions *($x \neq x'$* for which *$h(x)=h(x')$),* but would like it to be hard to find them.

Security definitions for hash functions

1. Weak collision resistance: for any $x \in X$, it is hard to find $x' \neq x$ such that $h(x)=h(x')$. (Also known as "universal one-way hash", or "*second* preimage resistance").
   - In other words, there is no efficient algorithm which given x can find an x' such that $h(x)=h(x')$.

2. Strong collision resistance: it is hard to find any x,x' for which $h(x)=h(x')$.
   - In other words, there is no efficient algorithm that can find a pair x,x' such that $h(x)=h(x')$.

Security definitions for hash functions

- It is easier to find collisions when you can choose both inputs.
  - In other words, under reasonable assumptions it holds that if it is possible to achieve security according to definition (2) then it is also possible to achieve security according to definition(1).
- Therefore strong collision resistance is a stronger assumption.

- Real world hash functions: MD5, SHA-1, SHA-256.
  - Output length is at least 160 bits.

Hmm..

The Birthday Phenomenon (Paradox)

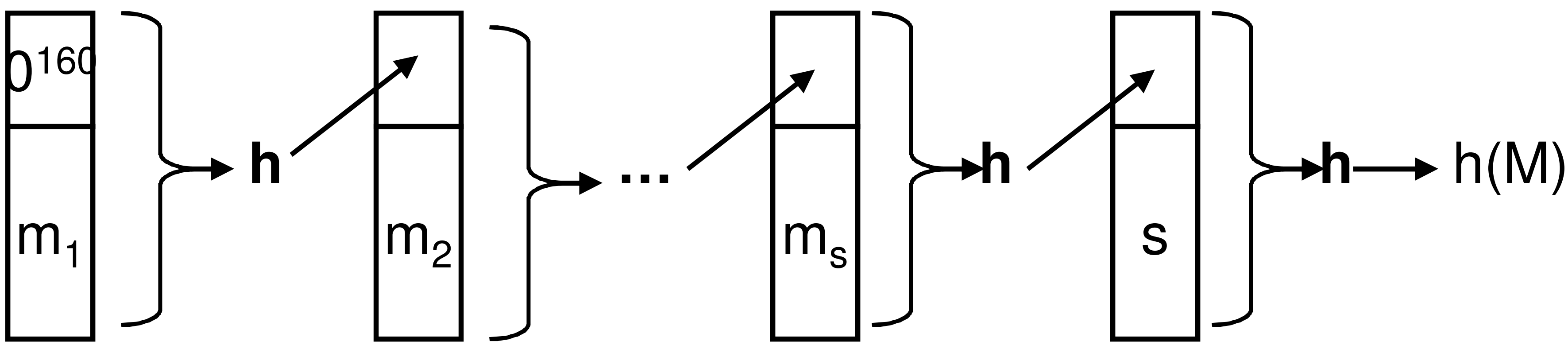
- For 23 people chosen at random, the probability that two of them have the same birthday is about ½.
- Compare to: The probability that one or more of them has the same birthday as Alan Turing is 23/365 (actually, $1-(1-1/365)^{23}$.)

- More generally, for a random $h:X \rightarrow Z$, if we choose about $|Z|^{½}$ elements of X at random ($1.17\ |Z|^{½}$), the probability that two of them are mapped to the same image is > ½.

- Implication: it's harder to achieve strong collision resistance
  - A random function with an n bit output
    - Can find x,x' with h(x)=h(x') after about $2^{n/2}$ tries.
    - Can find $x \neq 0$ s.t. h(x)=h(0) after about 2^n attempts.

From collision-resistance for fixed length inputs, to collision-resistance for arbitrary input lengths

- Hash function:
  - Input block length is usually 512 bits (|X|=512)
  - Output length is at least 160 bits (birthday attacks)
- Extending the domain to arbitrary inputs (Damgard-Merkle)
  - Suppose $h:\{0,1\}^{512} \rightarrow \{0,1\}^{160}$
  - Input: $M=m_1 \ldots m_s$, $|m_i|=512-160=352$. (what if $|M| \neq 352 \cdot i$ bits?)
  - Define: $y_0=0^{160}$. $y_i=h(y_{i-1},m_i)$. $y_{s+1}=h(y_s,s)$. $h(M)=y_{s+1}$.
  - Why is it secure? What about different length inputs?



Proof

- Show that if we can find $M \neq M'$ for which $H(M)=H(M')$, we can find blocks $m \neq m'$ for which $h(m)=h(m')$.

- Case 1: suppose $|M|=s$, $|M'|=s'$, and $s \neq s'$
  - Then, collision: $H(M)=h(y_s,s) = h(y_{s'},s')=H(M')$
- Case 2: $|M|=|M'|=s$
  - We know that $H(M)=h(y_s,s)=h(y'_s,s)=H(M')$
  - If $y_s \neq y'_s$ then we found a collision in h.
  - Otherwise, go from $i=s-1$ to $i=1$:
    - $y_{i+1} = y'_{i+1}$ implies $h(y_i,m_{i+1}) = h(y'_i,m'_{i+1})$.
    - If $y_i \neq y'_i$ or $m_{i+1} \neq m'_{i+1}$, then we found a collision.
    - $M \neq M'$ and therefore there is an i for which $m_{i+1} \neq m'_{i+1}$

The implication of collisions

- Given a hash function with 2^n possible outputs. Collisions can be found
  - after a search of $2^{n/2}$ values
  - even faster if the function is weak (MD5, SHA-1)

- We can find x, x' such that h(x)=h(x'), but we cannot control the value of x, x'.

- Can we find "meaningful" colliding values x, x' ?
  - The case of pdf/postscript files…

Basing MACs on Hash Functions

- Hash functions are not keyed. $MAC_K()$ uses a key.
- Best attack should not succeed with prob > $\max(2^{-|k|},2^{-|MAC()|})$.

- Idea: MAC combines message and a secret key, and hashes them with a collision resistant hash function.
  - E.g. $MAC_K(m) = h(k,m)$. (insecure.., given $MAC_K(m)$ can compute $MAC_K(m,|m|,m')$, if using the MD construction)
  - $MAC_K(m) = h(m,k)$. (insecure.., regardless of key length, use a birthday attack of $2^{|MAC()|\,/\,2}$ steps to find m,m' such that h(m)=h(m').)

Basing MACs on Hash Functions

- How should security be proved?:
  - Show that if MAC is insecure then so is hash function h.
  - Insecurity of MAC: adversary can generate $MAC_K(m)$ without knowing k.
  - Insecurity of h: adversary finds collisions ($x \neq x'$, $h(x)=h(x')$.)

HMAC

- Input: message *m,* a key *K*, and a hash function *h*.
- $HMAC_K(m) = h(K \oplus opad, h(K \oplus ipad, m))$
  – where ipad, opad are 64 byte long fixed strings
  – K is 64 byte long (if shorter, append 0s to get 64 bytes).
- Overhead: the same as that of applying h to m, plus an additional invocation to a short string.

- It was proven [BCK] that if HMAC is broken then either
  – h is not collision resistant (even when the initial block is random and secret), or
  – The output of h is not “unpredcitable” (when the initial block is random and secret)
- HMAC is used everywhere (SSL, IPSec).